# Linear Measurement in the Halacha<sup>1</sup>

### Prof. Avraham Greenfeld

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#### A. Introduction

There are two commonly accepted systems of equivalent values for Torah measurements, that of the Chazon Ish<sup>2</sup> and that of R. Chaim Naeh<sup>3</sup>. The Chazon Ish bases all halachic measures on the average width of a thumb (agudal) today, that is, 2.4 cm.; all other measurements are determined accordingly, e.g., the tefach (4 agudals), the amma (5 or 6 tefachs), the zeret

- 1. In 1982, I published an article entitled "Mida Keneged Mida" (Moria, 11:7-8) in which I cited the commonly accepted opinions concerning Torah measurements and their modern equivalents. I demonstrated the difficulties with each opinion, and I suggested a novel method for measuring the agudal, the thumb's-width, whereby it should be measured when placed on its side and not on its face. I demonstrated that that in this way, the greatest degree of correspondence is achieved among the different measures mentioned in Talmudic sources.
  - In the following issues of *Moria*, there appeared critical articles by R. Yaacov Kanievsky (*Moria*, 12:1-4), R. Kalman Kahana (*Moria*, 11:11-12), and R. Yedidia Manat (ibid.). In the present article, I shall present the main points of my original article (Hereafter: MKM) and my replies to my critics. I hope that publication of this article will foster and encourage further examination and discussion by Torah scholars of this important topic.
- 2. Chazon Ish, OH 39, "Kuntros HaShiurim"; R. Yaacov Kanievsky, Shiurin Shel Torah.
- B. R. Chaim Nach, Shiurei Torah; Shiurei Mikra; Shiurei Tzion.

(112 an amma), the revi'it (10.8 cubic agudals), etc. He states that even though in various places in Talmudic literature other methods are given for computing some of these measures, wherever a contradiction arises between two methods, "we should follow the agudal."

The contending opinion of Rav Nach asserts that we should accept without question the traditional equivalent values given for the different measures from the times of the Geonim and the Rambam. This tradition defines the *revi'it* as the equivalent of 86.4 cc., and thereby determines the *agudal* to be 2.0 cm. He believes that this value for the *agudal* is born out by modern measurements; however, even if there is a contradiction between tradition and modern measurements, we should rely on tradition.

Consequently, there is a 20% differential between the two measurements of the *agudal*. In volume measurements, this gives rise to a difference of up to 73%.

### B. The System of the Chazon Ish

The source of this opinion is the famous contradiction of the *Noda BeYehuda* (*Tzlach*, Pes. 116b), who perceived that the measure of *challa* determined as a multiple of eggs in his time was about half that reached as a multiple of the *agudal*. The *Noda BeYehuda* understood that both methods of calculation were dictated to Moshe by God (*Halacha LeMoshe MiSinai*) and therefore there could not be a contradiction between them. Since he could not imagine that the size of the human thumb had grown, he was compelled to conclude that the average egg of his time was only half that of Talmudic times, and ruled that in every case one should accept the measure that entails the stricter consequences. In theory, the Chazon Ish accepts the measurement based on the *agudal* even where the consequence is a leniency. The question however remains: How can two methods of measurement transmitted by the sages be so completely at odds?

The answer of the *Noda BeYehuda* is that the size of eggs has changed over the years. It must be noted that it is necessary according to this opinion to posit that not only the egg but an entire series of measurement standards have changed, as there are many other measurements that cannot be correlated with the *agudal*. The result is that we must assert that the *tefach* (the handbreadth) has shrunk, a cheekful and the volume of the gullet have shrunk, barley grains and carob seeds have grown lighter, and in general, many other physical

standards have changed; all this despite the statement of R. Sherira Gaon (quoted in *Sefer HaEshkol*, [Albeck] p.52) that measures were stated in terms of grains and fruits since they "are always available, so that one cannot claim that they have changed." Rav A.Y. Kook (notes to *Otzar HaGeonim*, Beiza p.61) comments: "The Gaon has stated explicitly that we should not worry about the possibility that there have been changes. I have on an authoritative source that eggs were recently found in Egyptian pyramids which were the same size as modern eggs. This constitutes persuasive support for the statement of R. Sherira Gaon."

Concerning possible changes in the human body, R. Moshe Feinstein (*Igrot Moshe*, YD III,66) states, "In reality, it is probable that there were no changes between the times of Moshe and the Sages and modern times." There exists no evidence from ancient skeletons that at one time men were significantly bigger or that some of their limbs were larger.<sup>5</sup>

Concerning possible changes in barley grains, R. Naeh demonstrates (*Shiur Mikva*, p.47) that the density of barley seeds today is equal to that of the Rambam, namely 0.73 that of water.<sup>6</sup>

Furthermore, the Geonim defined the five *selas* used in the redemption of the first-born as the weight of 1920 barley seeds (96 gram). They also define the weight of the water displacement of an egg as 1120 barley seeds. The Ramban and others state that the ratio of carob seeds to barley grains is 4:1. These ratios, between the egg, the carob seed, and the barley grain hold today as well. Hence, if the egg has shrunk by approximately 50%, we must conclude that the carob seed and barley grain have shrunk correspondingly. However, the measure used for the redemption of the first-born is the subject of an unbroken tradition and agrees with the smaller measurement. The Chazon Ish himself ruled that the measure for the redemption of the first-born should not be increased (*Shiurin Shel Torah*, pp.24–25). Since the the Chazon Ish maintained that the width of the barley seed has not changed (as seven widths

<sup>4.</sup> Prof. Y. Feliks, *Kelayei Zeraim Ve-Harkava*, p.184, n.5, writes: "I have examined the eggs discovered in the excavations at Pompeii. These also were the size of the small Arab egg of our times" (about 41.4 cc., as Feliks states subsequently).

<sup>5.</sup> The height of the average man has changed slightly, for instance from 160 cm. two hundred years ago to 171 cm. today, but these changes are natural and all parts of the body change proportionally.

<sup>6.</sup> He also cites (Shiurei Tzion, p.41, n.119) an expert in plant history to the effect that three thousand year old barley grains discovered in the pyramids are "exactly" equal to those today. Similarly, beans and other seeds found in Bet Shean, dating back 2500 years, were no larger than those of today.

of the barley seed are equivalent to the agudal), he was forced to conclude that their length and density had shrunk while their width had not.

The opinion of the the Chazon Ish leads to the conclusion that the average man of Talmudic times, while of the same height as today, was wider by 25%, had arms longer by 12 cm. (reaching below his knees), and had a thumb longer by 33%, thicker at its end, but nonetheless of the same width as today. All this is, at the least, rather fantastic, and unsupported by skeletal remains.

It is known today that selective breeding has natural limits.<sup>7</sup> For years, dairy farmers have been trying to achieve larger eggs, with the maximum achieved today about 75 cc. It is unlikely that there could ever be a situation where the average chicken egg would be 100 cc.

A different problem concerns the question when, according to this theory, did the change in the size of the egg occur. The Geonim (R. Hilai and R. Yehudai, quoted in Shiur Mikva p.27-31; R. Natronai, quoted in Shiurin Shel Torah, p.14), the Rambam (Commentary to the Mishna, Eduyot 1,2), and the author of the Shulchan Aruch (Avkat Rochel, 53) all define the measure of the revi'it as approximately 85 gram, which agrees with the modern egg. Numerous later authorities give a similar figure. The change must have taken place beforehand. This forces us to conclude that for a period of about 1000 years, until the time of the Noda BeYehuda, not one scholar bothered to measure the size of the egg and compare it with the agudal — including the Rambam (ibid.) and the Meiri (Pes. 109a) who explicitly state that they made volume measures based on the agudal. Is it conceivable that they did not compare the measure to the egg?

In conclusion: It is extremely difficult to accept an explanation based on the assumption that the size of the egg and other standards has changed.

### C. The System of R. Chaim Naeh

The system of R. Naeh, based on the value of 2.0 cm. for the agudal, founders on one basic problem. The Mishna (Kel. 17,6) states that Torah measures should be gauged "according to the understanding of the observer." R. Naeh explains that the estimation of the Rambam constitutes the "understanding of the observer" for our purposes. The Geonim however, emphasize that the standards of measurement were given in such a way as to

facilitate measurement by any person "according to his understanding", for all times and places.

R. Naeh claims that in fact there were no changes at any time. However, it is impossible to accept the figure of 2.0 cm. for the *agudal*. While it is true that he states that "exceptional skill and great sensitivity are necessary in order to determine the standard of an average person and an average thumb", measurements show an overwhelming majority of thumbs to be over 2.2 cm. wide.

### D. A New Method of Measuring the Agudal

#### 1. The Thickness of the Thumb

The usual method of measuring the thumb is across the width at the first joint. However, if we measure instead the thickness of the thumb (on its side) at the first joint, we find that all the measurement standards correspond without contradiction, and the results agree with the values given in medieval rabbinic literature.

The Tosafot (Men. 41b, s.v. "Arba'a") discuss whether to measure the agudal at the joint or at the end of the thumb. The Talmud (Zev. 63a) mentions measuring the thumb at its "zachrut". Rashi (according to the addenda of the Shita Mekubetzet, 13) comments, "The end ("rosh") of the thumb in its thickness ('ovee')." The Tosafot (Shita Mekubetzet, 32) explain that the zachrut is "the declination of the thickness ('ovee') of the thumb." Both Rashi and the Tosafot use the word ovee to explain measuring at the end of the thumb. Since the measurement at the end is across the thickness of the thumb, it stands to reason that the measurement at the joint is also across the thickness.

From the above Tosafot, it is apparent that the ratio of the agudal at the joint to the agudal at the end is 2:3. The same figure is mentioned by the Maharam of Rottenberg (Responsa, Prague ed., 239). If the thickness of the thumb is measured at the joint and at the end, the ratio is indeed about 2:3. However, if these measurements are taken across the width of the thumb, the ratio is closer to 1:2. This is a further proof of our contention.

The Migdal Oz (Hilchot Sefer Torah 9,10) writes: "The width of the pages of the Torah scroll which I wrote was equal to four fingers pressed together measured at the middle joint, which is slightly less than four agudals." This

comparison is only true if the *agudal* is based on the thickness of the thumb; if the width of the thumb is used, four fingers measured at the middle joint are equal to approximately three thumb's-widths. This too supports our contention.

#### 2. "Rochar Ha-Agudal"

The term "rochav ha-agudal", which would normally be translated as the width of the thumb, does appear frequently in the sources. However, I believe that here it means thickness, and should not be understood in opposition to the term "ovee ha-agudal". Let me explain.

In MKM, I explained that the term rochav can mean simply "the measure" of the thumb. I would like to suggest another possibility. Rashi (Zev. 63a) quotes an opinion that the agudal used to calculate the length of the ramp of the altar was measured at its end rather than at the joint. The Tosafot (Men. 41b) consider the possibility that this is the way that the agudal should always be measured. Accordingly, "rochav ha-agudal" may be understood as "the wide part of the thumb", in opposition to the end of the thumb. This is, in fact, explicit in the responsa of the Maharam of Rottenberg. The question posed was whether the agudal should be measured "from the middle in its rochav, above the joint, or below the joint." The Maharam answered that it is measured "at the rochav of the thumb at the middle... for at the end it is very narrow and sloping, and it is impossible to determine from where to measure. Rather, (it should be measured) at its rochav." It is clear that the term "rochav" here refers to a section of the thumb in opposition to the end; namely, the wider part. "Rochav" of the thumb" is synonymous with the joint. The Maharam concludes: "... it is apparent that it is measured from the middle of (the thumb) from the rochav of it."

The Mordechai has a similar usage of the term "rochav". "The estimation should be taken at the place of the rochav of the thumb and not at the end which is narrow." Clearly, "rochav" is used in contradistinction to "the end". Furthermore, in both this source and the Maharam, it is clear that the thickness of the end of the thumb is being measured ("very narrow and sloping"). Since the only difference between the two suggested methods of measurement is in the location of the measure, the thickness of the joint is the method that is being recommended.

Hence, all the sources quoted in the critical articles in *Moria*, which are all ultimately based on the Mordechai, do not contradict our premise. This is

explicitly stated by the *Pri Megadim* (OH 11, *Mishbetzot Zahav* 8): "Elsewhere, we have written that the Tosafot are undecided regarding this point (whether to measure from the joint or the end); however, we follow the opinion that we measure at the widest point. Wherever the *rochav* of the thumb is mentioned (it refers to) the upper joint at the widest point." He clearly defines the term "*rochav ha-agudal*" as the wide section of the thumb in opposition to the end, and not as the width in opposition to the thickness.<sup>8</sup>

In fact, in several early commentators, the term "ovee" (thickness) appears as often as the term "rochav", frequently together with it, without anyone perceiving a contradiction between the two. Either rochav means width and ovee means the wide point, or rochav means the wide point and ovee means thickness. The second possibility offers a better equivalent for "widest point", and has the added advantage of solving the problem of the incompatibility of the different standards of measurement.

In conclusion, there are three suggested interpretations for the term "rochav ha-agudal": (a) The width of the thumb — this is the understanding of all those late authorities who estimate the agudal to be 2.4 cm. This gives rise to the contradiction cited by the Noda BeYehuda; (b) The size of the agudal, without defining a particular place for the measurement — this is the interpretation I offered in MKM, p.64. This interpretation was disputed by my critics in the succeeding issues of Moria, although, I believe, not convincingly; (c) The widest point of the thumb — this is the interpretation presented in the present article.

Another reason why the thickness of the thumb might be called *rochav* is apparent if the hand is laid flat, palm down, especially if the fingers are closed. In this position, the thumb is lying on its thickness while the other four fingers on on their widths. It is possible that the thickness of the thumb, since it appears together with the width of the other fingers, is called *rochav haagudal*.

The first to clearly mention measuring the width of the thumb is the *Tevuot Shor* (*Simla Chadasha*, YD 35,38). It is noteworthy that his statement was published only fifty years before the *Noda BeYehuda* discovered the contradiction between the *agudal* and the measure of the egg. Before this

<sup>8.</sup> Cf. Maharsham, *Daat Torah*, YD 35,116: "'In its *rochar*' means in the middle of the wide part of the thumb."

<sup>9.</sup> Or Zarua, Hilchot Treifot, p.57: Hagahot Ashri, Chul. 3.16: Responsa Raavad 6: Terumat HaDeshen 1.74: Maharil, Hilchot Bedika, p.80: Hilchot Uminhagei Maharash 483. Some of these sources were located through the Computerized Responsa Project of Bar Ilan University.

period, the accepted value for the *revi'it* was about 60 cc. (see *Shiur Mikva*, pp.62-63). Not only did Torah scholars not estimate the measure based on the *agudal* to be larger than that based on the egg, they claimed, based on a visual estimate alone, that 10.8 cubic *agudals*, which according to the Talmud (Pes. 109a) is equal to the volume of an egg and a half, appears to be less than one egg.10

Rav Manat, in his article in *Moria*, cites numerous sources requiring the use of *rochav ha-agudal*, which he believes can only mean the width. Aside from the alternative interpretations offered above, of which one or the other is appropriate in every source which he cites, it is worth noting that, in many cases, rabbinic terminology treats *rochav* and *ovee* as synonyms. See, for example, Rashi (Ezek. 40,5), who explains the biblical "*rochav* of the building" as "the *ovee* of the wall." R. Chaim Benish has pointed out to me that the Rambam often interchanges the two terms. See, for example, the Commentary to the Mishna (Ohal. 16,1) and *Hilchot Tum'at Meit* (12,5), as well as his references to the thickness of a wall (*Hilchot Shecheinim* 2,15, 2,18, *Hilchot Shabbat* 28,14).

#### 3. Practicality of Measuring

It is worth noting that it is extremely difficult to measure distances using the width of the thumb. Most people would find it difficult to lay their thumb flat in the middle of a surface. Using the thickness of the thumb is, to the contrary, very convenient. It is relatively easy to lay one thumb next to the other in this manner and thus measure any distance. (This is done by reversing the direction of opposite hands; i.e, the tip of the left thumb is placed against the base of the right thumb, and vice versa, keeping the joints on a straight line.)

## D. Correspondence of Different Standards

The following table lists the relationships between the different standards based on the human body and the *agudal*, measured in its width and in its thickness. The values used were:

Thickness of the thumb: 1.95 cm.

Width of the thumb: 2.4 cm.

Thickness of the end of the thumb: 1.3 cm.

10. Responsa Chut HaShani 97. The work was published approximately 100 years before the Tzlach. A similar statement appears in the Siddur of R. Yaacov Emden (Hanhagat Seuda 14,8).

Standard	Measured Value (cm.)	Ratio to width of agudal	Ratio to thickness of agudal	Ratio According to Sages
tefach	7.8	3.3	4.0	4
zeret	19	7.9	9.7	10
(amma = 5 tefach)				
zeret	22.5	9.4	11.5	12
amma = 6 tefach				
amma (5 tefach)	37.5	15.6	19.2	20
amma (6 tefach)	46	19.2	23.6	24
thickness of the end	1.3	0.54	0.67	0.67
of thumb				

As can be seen from the table, there is a high degree of correspondence between the ratio prescribed by the Talmudic sources (column four) and the values reached using the thickness of the thumb. The values reached using the width of the thumb diverge consistently from the expected values by approximately 20%.

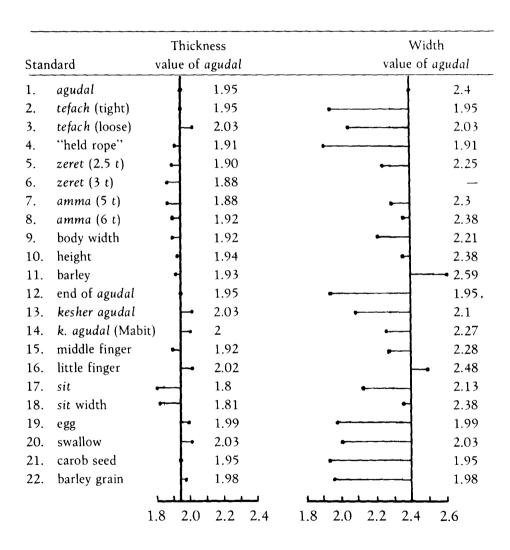
The following table and graph, containing 22 different measurement standards, demonstrates the correspondence of our method of measuring the agudal with the other measurement standards. These include linear, volume, and weight measures. For each item, the ratio to the agudal given in Talmudic literature is listed, followed by the average value using the interpretation most appropriate for each system (thickness and width), as explained after the tables. The last value is divided by the Talmudic ratio to give a corresponding value for the agudal. Ideally, this value should agree exactly with the base value for the agudal in each system. The discrepancies are shown graphically in the second table.

	Ratio According	Average Value According to Thickness Width Thickness Width		Value of the Agudal According to	
Standard	to				
	the Sages				
	length	(in cr	n.)		
1. agudal	1	1.95	2.4	1.95	2.4
2. <i>tefach</i> (tight)	4	7.8	7.8	1.95	1.95
3. tefach (loose)	4.08	8.3	8.3	2.03	2.03
4. "held rope"	4.5	8.6	8.6	1.91	1.91
5. zeret (2.5 t)	10	19	22.5	1.9	2.25
6. zeret (3 t)	12	22.5	nonexistent	1.88	-
7. amma (5 t)	20	37.5	46	1.88	2.3
8. amma (6 t)	24	46	57	1.92	2.38
). body width	24	46	53	1.92	2.21
10. height	72	140	171	1.94	2.38
11. barley	1/7	0.275	0.37	1.93	2.59
12. end of agudal	2/3	1.3	1.3	1.95	1.95
l·3. kesher agudal	3	6.1	6.3	2.03	2.1
l 4. k agudal (Mabit)	1.5	3	3.4	2	2.27
15. middle finger	5/6	1.6	1.	1.92	2.28
16. little finger	2/3	1.35	1.65	2.02	2.48
7. sit	4	7.2	8.5	1.8	2.13
18. sit width	8	14.5	19	1.81	2.38
	volume	e (in c	c. )		
19. egg	3√7.2	57	57	1.99	1.99
20. swallow	3√7.2	60	60	2.03	2.03
	weight	(ing	r. )		
21. carob seed	$3\sqrt{280} \times 7.2$	0.19	0.19	.95	1.95
22. barley grain	$3\sqrt{1120}\times7.2$	0.05	0.05	1.98	1.98

The graph below displays the divergence of each value appearing in the last two columns of the above table from the base value of the *agudal* in each system. The vertical line is set for 1.95 cm. in the first case and 2.4 cm. in the second. The horizontal lines indicate the divergence for each item, based on the scale at the bottom of the graph.

The graph demonstrates clearly that the divergences present in the system based on the thickness of the thumb are far less than in the traditional system

based on the width of the thumb. In fact, the divergences in the latter case would have been far greater had we not accepted several doubtful definitions of the various measures solely in order to achieve as close a correspondence as possible. (See, for example, 8.2 in the explanations after the graph.)



### Explanation of the Table

For each numbered item, the method of calculation appropriate to the thickness system is subnumbered 1; the method appropriate to the width method is subnumbered 2.

- 1.1 agudal. The diameter of the thickness of the thumb at the first joint.
- 1.2 The diameter of the width of the thumb at the first joint.
- 2. *tefach* (tight *otzev*). The width of a fist measured across the knuckles, with the fingers tightly pressed together.
- 3. tefach (loose sochek). Same as 2, but with the fingers held loosely.
- 4. "held rope". The length of a rope enclosed in a closed fist (where the thumb is pressed against the side of the adjacent finger).
- 5.1 *zeret* (2.5 *tefach*). The distance between the thumb and the small finger, with the thumb spread far from the hand and the other fingers pressed together.
- 5.2 Same as 5.1, but with all the fingers spread wide.
- 6.1 *zeret* (3 *tefach*). Same as 5.2
- 6.2 Nonexistent
- 7.1 *amma* (5 *tefach*). The distance from the elbow to the knuckles.
- 7.2 The distance between the elbow and the tip of the middle finger. This method was suggested in *Shiurin Shel Torah* (6,7). It is specifically contradicted by several early authorities.
- 8.1 *amma* (6 *tefach*). The distance between the elbow and the tip of the middle finger.
- 8.2 The distance between the shoulder and the wrist joint. This method is derived from the Maharitz (*Bet Shaul*, cited in *Shiur Mikva* p. 138), but is contradicted by several early authorities.
- 9.1 **body width**. The outer distance between the arms (at the shoulder) without much clothing (Tosafot, Pes. 109b, Yoma 31b; Ritva, BB 101b).
- 9.2 The outer distance between the arms (at the elbows) with much clothing (Ramban, BB 101b; Ran, ad.loc.).
- 10.1 height. Shoulder height (naked).
- 10.2 Height including the head (naked).
- 11.1 barley. The thickness of a barley grain, including chaff.
- 11.2 The width of a barley grain, including chaff.
- end of *agudal*. The thicknes of the tip of the thumb, at the center of the nail.

- 13.1 **kesher agudal**. The length of the thumb, from the tip to the second joint. Rabbenu Gershon (Men. 42a), the Rid (*Hilchot Tzitzit* ch. 3), and the Riaz (ibid.) state that this is the equivalent of three agudals.
- 13.2 The length of the thumb from the tip to the end of the second joint.
- 14.1 **kesher agudal** (Mabit). The distance from the base of the nail to the middle of the first joint. The Mabit (*Responsa* 143) states that this is equivalent to 1.5 agudals.
- 14.2 The distance from the tip of the thumb to the base of the first joint.
- 15.1 middle finger. Thickness at the upper joint.
- 15.2 Width at the upper joint.
- 16.1 little finger. Thickness at the upper joint.
- 16.2 Width at the upper joint.
- 17.1 sit. Inner distance between the forefinger and the middle finger.
- 17.2 The distance between the ends of the forefinger and the middle finger. The Rach (Shab. 106) and the *Aruch* ("*kaf*") are the only authorities who suggest measuring from the ends of the fingers, in contradistinction to all other sources, who state that the measurement should be taken "between" the fingers.
- 18.1 *sit* width (Rambam). The inner distance between the forefinger and the thumb when spread as far as posssible, and the hand is resting on a level surface.
- 18.2 Same as 18.1, but measured from the ends of the fingers; cf. 17.2.
- 19 egg. The volume of water displaced by a standard (but not average) egg.
- 20 **swallow**. The volume of food in a single swallow.
- carob seed. Weight in grams, equivalent of 4 barley grains (Shiurin Shel Torah, pp.24-25).
- barley grain. According to Geonim, the weight of 1120 barley grains is equal to the weight of an eggs volume of water (Shiurin Shel Torah, ibid).

The graph demonstrates that for nineteen out of twenty-one standards, the compatibility with the thickness of the thumb as the basis for the *agudal* is between 96-100%. The two remaining cases (17,18) evince a compatibility of 92%. Both of thious measurements from the animal and vegetable world, is very high.

Using the width of the thumb as a basis for the *agudal*, on the other hand, results in a similar degree of compatibility (96-100%) for only five standards! For most of the other standards, the values differ widely from the base value. It is hard to ignore these results.